Prespecified-Time Control of Complex Networks Coupled with Nonlinear Dynamical Systems

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ABSTRACT

The control of complex networks is a hot-spot topic in the field of complex networks. Based on Lyapunov function and matrix theory, a control scheme for complex networks coupled with nonlinear dynamical systems is presented here. Different from the existing finite-time control strategies, the settling time of our proposed scheme does not depend on initial values or the control parameters of the system, and can be given arbitrarily. In addition, the scheme is applicable to both directed and undirected networks, and connectivity is not required. Finally, two simulations are provided to confirm the feasibility of our theoretical result.

CCS CONCEPTS

• **Networks**; • **Network protocols**; • **Network protocol design**;

KEYWORDS

Prespecified-time control, Complex networks, Time-scaling technique

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1 INTRODUCTION

Due to the development of network information technology, human society has entered the era of complex networks [1-3]. Generally, a complex network can be described as many nodes and edges. When two nodes have a certain relationship between them, they connect an edge, otherwise, no edges are connected. Two nodes connected by edge is regarded as adjacent in the network. For example, the metabolic network can be seen as a network connected by enzymes through metabolites [4]; Social network can be regarded as a network formed by a variety of relationships, such as friendship, cooperation and business relationships [5, 6]. Aviation network can be regarded as a network formed by setting up flights in different cities [7]; There also exists power grid networks [8], military networks [9] and disease networks [10]. It is obvious that complex networks are everywhere and closely related to human life.

On the one hand, the ultimate goal for complex networks is to control the dynamic behavior. The structural controllability of complex dynamical systems started from Liu [11], who studied from the perspective of linear cybernetics. Drived by this research, the structural controllability has become a hot topic in network science, and a large number of research works have emerged successively [12-15]. Later, Yuan et al. solved the problems of strict controllability and control input of complex networks [16], which broadened its application scope. Whether it is structural control or strict control, it is only for linearly coupled complex network dynamic systems.

However, in reality, there are a variety of nonlinear systems, which have more complex dynamic behaviors. Recently, biological neural networks have attracted the attention of scientific researchers. As the basis of the nervous system that generates cognitive function, biological neurons play an incomparable role in information optimization and processing, understanding the memory rules of the brain and other aspects. At present, the biological neural network is the most complex nonlinear dynamic system ever discovered. And the in-depth study of its nonlinear dynamical system is of great theoretical value for revealing the process, cognition and thinking mechanism of brain neural information transmission. In 1982, Hindmarsh and Rose proposed the HR model. Because it can describe all kinds of neural activities observed in the real neuron system, the model has been widely used in the dynamic analysis of neurons and the synchronization control of nonlinear dynamical system. Therefore, the study of nonlinearly system is an important bridge for the real application of complex networks.

On the other hand, in the evaluation index of the control system, the convergence performance is a very key index. Thus, a striking extension of finite-time control methods has appeared [17-19]. In [17], a continuous finite time state feedback controller is proposed for a kind of double integral system for the first time. Using finite time control technology in [18], a continuous controller is developed to solve the synchronization problem of two chaotic systems. A finite-time control method is proposed for both traditional and overlapping cluster networks in [19]. Some more finite-time control method can be seen in [20-21–23].

It should be noted that the settling time of finite-time control depends heavily on the initial system values and controller parameters. However, in some mechanical processes or industrial applications, it is not easy to satisfy either of these conditions [24]. Therefore, it is of great significance to design a finite time control strategy

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which can set the stability time in advance. Recently, some scholars have proposed the prespecified-time control strategy [25-29]. By building a novel scaling function, [25] ensured the multi-agent systems reach prescribed-time consensus under a spanning tree of directed topology. Chen et al. [27] put forward prespecified-time decentralized regulation protocols for strict feedback nonlinear multi-agent systems. And [29] focused on the prescribed-time cluster synchronization with average dwell-time constraint switched signal complex networks.

Therefore, it is of great theoretical and practical significance to design a control strategy for nonlinear dynamical system to achieve stability in a prespecified time. Based on the above analysis, this paper put forward a newly control strategy, which is applicable to both undirected network and directed network, and does not need to satisfy connectivity.

The highlights of this paper are as follows:

- We know that the settling time in finite-time control is related to initial values and the control parameters, while in our proposed one, it can be user-specified through the time-varying function ω(t).
- By regarding the desired orbit as a virtual system and constructing a newly graph, the original network does not need to satisfies symmetry and connectivity. Thus the control scheme we proposed can work in both undirected and directed networks.

2 PRELIMINARIES

Let $G = (v, E)$ be a directed/undirected graph. The adjacent matrix $A = (a_{ij}^{ad})_{m\times m}$ is defined as $a_{ij}^{ad} = 1$for $i \neq j, i, j \in v = \{1, 2, \cdot sN\}$if there exists a path from node $j$to node i, otherwise $a_{ij}^{ad} = 0$. And $D = diag\{d_1, d_2, \cdot sd_M\}$ is the degree matrix of the graph G with d_i is the in-degree of vertex i. The Laplacian matrix is defined as $l_{ij} < 0$if there is an edge from node$j$to node$i$, otherwise$l_{ij} = 0$. Besides$l_{ii} = -\sum_{j=1, j\neq i}^{N} l_{ij}$for all$i$. Denote by 1_N a column vector with all elements equal to 1.

Assumption 1: For all u, $v \in \mathbb{R}^m$, there exist a positive constant ρ such that

$$[u - v]^T [f(u) - f(v)] \leq \rho[u - v]^T [u - v] \tag{1}$$

Lemma 1[25]: If a graph G contains a directed spanning tree, then the Laplacian matrix L is given by

$$L = \begin{bmatrix} L_1 & L_2 \\ 0_{1\times(N-1)} & 0_{1\times1} \end{bmatrix} \tag{2}$$

where L_1 is a nonsingular M-matrix. Additionally, a positive matrix$K$is given by$K = diag[k_1, k_2, \cdot sk_{N-1}]$, which satisfies

$$Q = L_1 K + K L_1{}^T > 0 \tag{3}$$

Definition 1[25]: The system is said to reach prespecified-time synchronization if and only if there exists a presetting constant $T_p$which holds $u_i(t) \to u_s(t)$as$t \to T_p$,$u_i(t) = u_s(t)$ as t≥T_p。

Definition 2[24]: Consider the time-varying function $\omega(t)$,$\omega(t) = \frac{T_p}{T_p+t_n-t}$when$t \in [t_n, t_{n+1})$,where$T_p = t_{n+1} - t_n$ is a finite positive constant giving by users, n=0,1,2, ...

It can be seen that this function has different function expressions at different time periods and equal spacing T_p.

3 MAIN RESULTS

The network of N nodes is considered as

$$\dot{u}_i(t) = f(u_i(t)) + c\sum_{j\in v} a_{ij}^{ad}(u_j(t) - u_i(t)) + \Phi_i(t) \tag{4}$$

which holds for all i $\in v$, where$u_i(t)$ is the state of node i, fis a continuous function, $c > 0$ is the coupling strength, $\Phi_i(t)$ is the control input and to be designed.

$u_s(t)$is the target trajectory which satisfies

$$\dot{u}_s(t) = f(u_s(t)) \tag{5}$$

For all i $\in v$, the controller is presented as

$$\begin{aligned} \Phi_i(t) = &\frac{h}{T_p}\omega(t) \sum_{j\in v} a_{ij}^{ad}(u_j(t) - u_i(t)) \\ &- (c + \frac{h}{T_p}\omega(t))(u_i(t) - u_s(t)) \end{aligned} \tag{6}$$

where h is a positive constant. Then we have

$$\begin{aligned} \dot{u}_i(t) = f(u_i(t)) + c\sum_{j\in v} a_{ij}^{ad}(u_j(t) - u_i(t)) \ = f(u_i(t)) - c\sum_{j\in v} l_{ij}u_j(t) \\ + \frac{h}{T_p}\omega(t)\sum_{j\in v} a_{ij}^{ad}(u_j(t) - u_i(t)) - \frac{h}{T_p}\omega(t)\sum_{j\in v} l_{ij}u_j(t) \\ - (c + \frac{h}{T_p}\omega(t))(u_i(t) - u_s(t)) - (c + \frac{h}{T_p}\omega(t))(u_i(t) - u_s(t)) \end{aligned} \tag{7}$$

Setting $u_{N+1}(t) = u_s(t)$, and we regard the desired target orbit as a virtual system, further

$$\begin{aligned} \dot{u}_i(t) = &f(u_i(t)) - c\sum_{j=1}^{N+1} \bar{l}_{ij}u_j(t) - \frac{h}{T_p}\omega(t)\sum_{j=1}^{N+1} \bar{l}_{ij}u_j(t) \\ &= f(u_i(t)) - c\sum_{j\in v} \bar{l}_{ij}(u_j(t) - u_s(t)) \\ &- \frac{h}{T_p}\omega(t)\sum_{j\in v} \bar{l}_{ij}(u_j(t) - u_s(t)) \end{aligned} \tag{8}$$

where $\bar{L}$ is related to L.

$\bar{L} = \bar{l}_{ij} = [\begin{matrix} L+I & I \\ 0 & 0 \end{matrix}]$.And let $L1$=L+I.

Remark 1: *It is precisely because we regard the desired target orbit as a virtual system and construct a new connected graph that the original network itself does not need to satisfy the connectivity.*

Denote the error system as

$$\varepsilon_i = u_i - u_s \tag{9}$$

Then we have

$$\begin{aligned} \dot{\varepsilon}_i(t) = &f(u_i(t)) - f(u_s(t)) - c\sum_{j\in v} \bar{l}_{ij}\varepsilon_j(t) \\ &- \frac{h}{T_p}\omega(t)\sum_{j\in v} \bar{l}_{ij}\varepsilon_j(t) \end{aligned} \tag{10}$$

which can be written as the following matrix form

$$\begin{aligned} &\varepsilon = F(u) - F(s) - c(L_1 \otimes I_m)\varepsilon \\ &- \frac{h}{T_p}\omega(t)(L_1 \otimes I_m)\varepsilon \end{aligned} \tag{11}$$

whereε= (ε_1(t),ε_2(t), $\cdot\, s\varepsilon_N$(t)),$F(u) = (f(u_1(t), f(u_2(t), \cdot sf(u_N(t)))$, $F(s) = (f(s(t), f(s(t), \cdot sf(s(t)))$.

Remark 2: *It should be noted that the convergence time in finite-time and fixed-time control can only be bounded in a finite time period. Here, in our proposed prespecified-time control scheme, the* *convergence time can be preassign it arbitrarily as needed without considering the system's behaviors.*

Theorem 1. Under Assumption 1, network (4) can reach global prespecified-time synchronization by using the proposed controller (6), provided that

$$\frac{\lambda_{\min}(Q)}{\lambda_{\max}(K)} > 0 \tag{12}$$

$$c\frac{\lambda_{\min}(Q)}{\lambda_{\max}(K)} - 2\rho > 0 \tag{13}$$

Then the synchronization can be achieved in the finite presetting time T_p and remain unchanged over $[t_1, \infty)$..

Proof: we select the Lyapunov function as

$$V = \varepsilon^T (K \otimes I_m)\varepsilon \tag{14}$$

And take the derivative of V,

$$\begin{aligned} \dot{V} &= 2\varepsilon^T (K \otimes I_m)\dot{\varepsilon} \\ &= 2\varepsilon^T (K \otimes I_m)[F(u) - F(s)] \\ &\quad - 2c\varepsilon^T (KL_1 \otimes I_m)\varepsilon \\ &\quad - 2\frac{h}{T_p}\omega(t)\varepsilon^T (KL_1 \otimes I_m)\varepsilon \\ &\leq 2\rho\varepsilon^T (K \otimes I_m)\varepsilon \\ &\quad - c\varepsilon^T [(KL_1 + L_1{}^T K) \otimes I_m]\varepsilon \\ &\quad - \frac{h}{T_p}\omega(t)\varepsilon^T [(KL_1 + L_1{}^T K) \otimes I_m]\varepsilon \end{aligned} \tag{15}$$

According to Lemma 1, there exists a positive matrix $K$and satisfies

$$Q = KL_1 + L_1{}^T K > 0 \tag{16}$$

where $K = diag[k_1, k_2, \cdot sk_M]$. Then we have

$$\begin{aligned} \dot{V}(t) &\leq 2\rho\varepsilon^T (K \otimes I_m)\varepsilon - c\frac{\lambda_{\min}(Q)}{\lambda_{\max}(K)}\varepsilon^T (K \otimes I_m)\varepsilon \\ &\quad - \frac{h}{T_p}\omega(t)\frac{\lambda_{\min}(Q)}{\lambda_{\max}(K)}\varepsilon^T (K \otimes I_m)\varepsilon \end{aligned} \tag{17}$$

Let $\xi = c\frac{\lambda_{\min}(Q)}{\lambda_{\max}(K)} - 2\rho > 0, \delta = \frac{\lambda_{\min}(Q)}{\lambda_{\max}(K)} > 0$.

Further

$$\dot{V}(t) \leq -\xi V(t) - \delta\frac{h}{T_p}\omega(t)V(t) \tag{18}$$

Denote $\psi(t) = \omega^m(t)$,then we have $\dot{\psi}(t) = m\omega^{m-1}(t)\dot{\omega}(t)$,

Thus

$$\frac{\dot{\psi}(t)}{\psi(t)} = \frac{m\dot{\omega}(t)}{\omega(t)} = \frac{m\omega(t)}{T_p}.$$

and multiply both sides of inequality (18) by $\psi^\delta(t)$

$$\psi^\delta(t)\dot{V}(t) \leq -\xi\psi^\delta(t)V(t) - \delta\frac{h}{T_p}\omega(t)\psi^\delta(t)V(t) \tag{19}$$

Case 1: $t \in [t_0, t_1)$. From (19) ones have

$$\begin{aligned} (\psi^\delta(t)V(t))' &\leq -\xi\psi^\delta(t)V(t) \\ \psi^\delta(t)V(t) &\leq e^{-\xi(t-t_0)}\psi^\delta(t)V(t_0) \\ V(t) &\leq e^{-\xi(t-t_0)}\omega^{-m\delta}(t)\omega^{m\delta}(t_0)V(t_0) \end{aligned} \tag{20}$$

From the definition of $\omega(t)$, we have

$$\lim_{t \to t_0^+} \omega^{-m\delta}(t) = 1, \lim_{t \to t_1^-} \omega^{-m\delta}(t) = 0. \tag{21}$$

Further we have

$$V(t) \leq e^{-\xi(t-t_0)}\omega^{-m\delta}(t)V(t_0) \tag{22}$$

Notice that$\omega^{-m\delta}(t) \to 0$, as$t \to t_1^-$.And then

$$\lim_{t \to t_1{}^-} \parallel \varepsilon(t) \parallel \to 0 \tag{23}$$

Case 2: The system is maintained stable over $[t_1, \infty)$.

Since V(t) is differentiable, then it is continuous, which further yields $V(t_1) = \lim_{t \to t_1^-} V(t) = 0$.From inequality (18) we have V(t) is monotonically decreasing. Besides,

$$0 \leq V(t) \leq V(t_1) = 0, [t_1, t_2). \tag{24}$$

That is, $V(t) \equiv 0, \varepsilon(t) \equiv 0on[t_1, t_2)$. And by induction, $\varepsilon(t) \equiv 0$on$[t_n, t_{n+1})$. Then we have $\varepsilon(t) \equiv 0$on$[t_1, \infty)$. It can be seen that through prespecified-time control, the system can achieve synchronization within a finite time $\mathrm{T_p}$.

Remark 3: *In some mechanical processes or industrial applications, the settling time must be chosen to drive the system state to a given precision. Therefore, the control strategy that can obtain the stability time in advance is of great significance.*

4 NUMERICAL SIMULATIONS

In this part, two numerical examples are provided to demonstrate the feasibility of the proposed scheme. Holden et al. proposed the HR model in 1992, which can describe the numerical calculation of the firing behavior of neurons. Since then, many researchers have used the HR neuron system model to carry out a lot of experiments on the nonlinear dynamics of the nervous system. Now, HR systems are widely used to simulate the behavior of real biological neurons [25]. Here we use single HR neuron system as node dynamics, which is described by:

$$\begin{cases} \dot{x} = \alpha x^2 - x^3 - y - z \\ \dot{y} = (\alpha + d)x^2 - y \\ \dot{z} = r(\beta x + b - z) \end{cases} \tag{25}$$

where$\alpha = 2.8, d = 1.6, r = 0.001, \beta = 9, b = 5$.The fast membrane voltage is expressed by x, the recovery variable was expressed by y, and the slow adaptation current was expressed by z.

By setting the appropriate parameters, the HR neuron system shows periodic discharge changes, as shown in Figure 1

4.1 The Network with Undirected Topology

First we examine the undirected graph, whose Laplacian matrix is as follows

$$L_u = \begin{pmatrix} 2 & -1 & 0 & 0 & -1 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 \\ -1 & -1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix}$$

Then we choose the coupling strength c=0.8, the control parameter h=1 and let the convergence time T_p=4*s*. The initial values of each state are set randomly among [0,1]. And the evolutions of control input for each dimension is shown in Figure 2

The results shown in Figure 3 illustrate that the nonlinear dynamical HR neuron system achieve synchronization within a prespecified finite time T_p=4*s*.

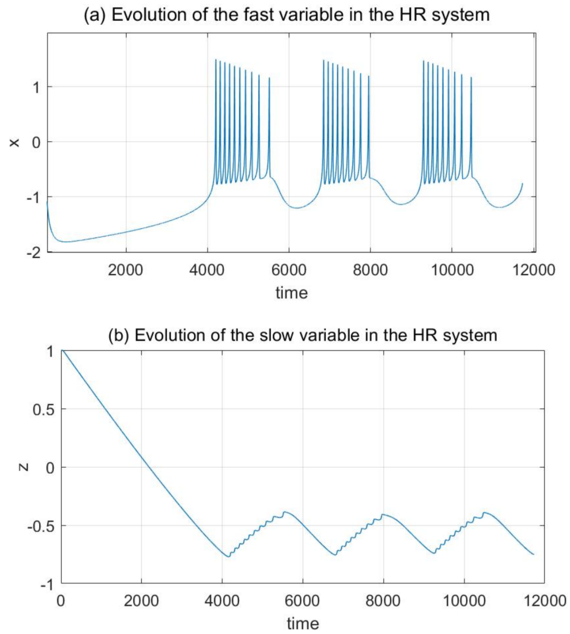

Figure 1: Evolution of the Fast (A) and Slow (B) Variables in the Hr Neuron System.

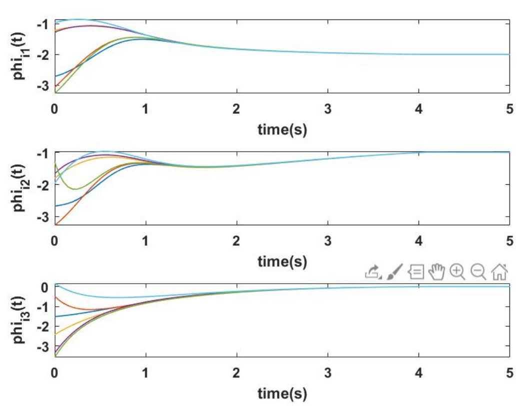

Figure 2: Evolution of Control Inputs of Undirected Graph.

4.2 The Network with Directed Topology

Then we examine the directed graph, whose Laplacian matrix and topology are as follows.

$$L_d = \begin{pmatrix} 4 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

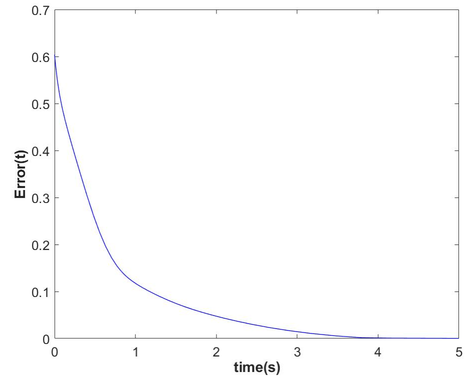

Figure 3: Evolution of Total Errors of Undirected Graph.

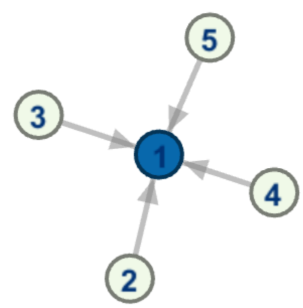

Figure 4: The Topology of Directed Graph.

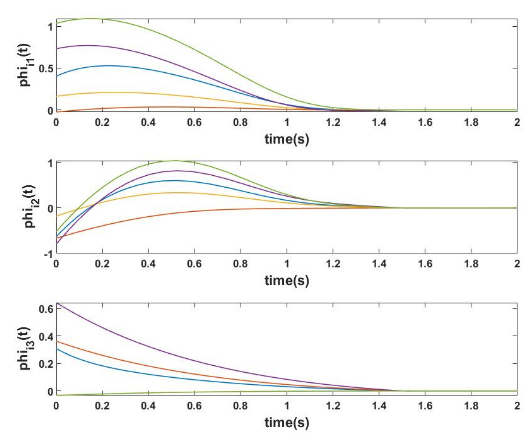

Figure 5: Evolution of Control Inputs of Directed Graph.

From Figure 4 we see that the directed graph is unconnected. Here, we choose $c = 1$, $h = 1$ and $T(p) = 1.5s$ The evolution of errors is shown in Figure 5, which depicts that our control method is suitable for disconnected topology. From Figure 2 and Figure 6 we can see that the control input is zero after the prespecified-time T_p and is not be too large.

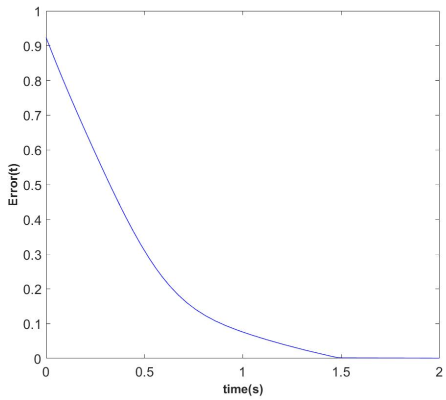

Figure 6: Evolution of Total Errors of Directed Graph.

5 CONCLUSION

In this paper, we investigate the prescribed-time control of complex networks coupled with nonlinear dynamical system. The convergence time can be preassigned arbitrarily which is different from finite-time control. And the control scheme we proposed is also works in disconnected graph. Our future research direction is to consider time delays and data dropout during information transmission.

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